Operational Game Semantics (OGS) formally: Let's talk about chattering

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A bit of context

Original motivation: interactive semantics for FFI

Step 1: use OGS, prove soundness…

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… this is actually kind of tricky?! (me, one year in)

$$\frac{[\![p]\!]_{\mathrm{OGS}} \approx [\![q]\!]_{\mathrm{OGS}}}{\forall E, \gamma,\ \mathrm{eval}(E[p[\gamma]]) \equiv_{\mathrm{res}} \mathrm{eval}(E[q[\gamma]])}\ \text{soundness}$$

Formalization challenges

Syntax and operational semantics are tedious.

$\Rightarrow$ Just enough precision (not more).

OGS requires subtle coinductive reasoning.

$\Rightarrow$ Answer in this talk.

Key choices

Traces in intrinsically typed and scoped De-Bruijn.

Axiomatize what makes OGS sound.

Copattern- and coalgebra-based presentation.

Outline

1. Our flavor of Operational game semantics.
2. Composition and the mystery hypothesis.
3. Concluding with eventual guardedness.

Our flavor of Operational Game Semantics

Observations*

What can you ask to a...

function?	$\cdot\mathtt{app}(v, \kappa)$
pair?	$\cdot\mathtt{fst}(\kappa)$, $\cdot\mathtt{snd}(\kappa)$
stream?	$\cdot\mathtt{head}(\kappa)$, $\cdot\mathtt{tail}(\kappa)$
continuation?	$\cdot\mathtt{ret}(v)$

* also called “copattern”

In practice

Partiality: the delay monad

$$\mathcal{D}(X) \coloneqq \nu A.\, A + X$$

Operational semantics: sequent calculus-style

$$\mathsf{Val} \colon \mathsf{Ctx} \to \mathsf{Typ} \to \mathsf{Set} \qquad \mathsf{Conf} \colon \mathsf{Ctx} \to \mathsf{Set} \qquad \mathsf{Obs} \colon \mathsf{Typ} \to \mathsf{Set}$$

$$\mathsf{eval} \colon \forall \Gamma, \mathsf{Conf}\, \Gamma \to \mathcal{D}(\mathsf{Nf}\, \Gamma) \qquad \mathsf{holes} \colon \mathsf{Obs}\, \tau \to \mathsf{Ctx}$$

$$\mathsf{Nf}\, \Gamma \coloneqq (x \colon \tau \in \Gamma) \times (o \colon \mathsf{Obs}\, \tau) \times (\gamma \colon \mathsf{holes}(o) \to_{\mathsf{Val}} \Gamma)$$

$$c \approx_{\mathsf{ctx}} d \coloneqq \forall \gamma, \mathsf{eval}\ (c[\gamma]) \equiv_{\mathsf{Obs}} \mathsf{eval}\ (d[\gamma])$$

Strategies: Set families S^{+}, S^{-}, equipped with:

$$\mathrm{play} \colon S^{+}\ (\Gamma, \Delta) \to \mathcal{D}\left((o \colon \mathsf{Obs}^{\bullet}\ \Gamma) \times S^{-}\ (\Gamma, \Delta + \mathsf{holes}(o))\right)$$
$$\mathrm{coplay} \colon S^{-}\ (\Gamma, \Delta) \to (o \colon \mathsf{Obs}^{\bullet}\ \Delta) \to S^{+}\ (\Gamma + \mathsf{holes}(o), \Delta)$$

Strategies: Set families S^+, S^-, equipped with:

$$\mathsf{play}\colon S^+\;(\Gamma,\Delta) \to \mathcal{D}\left((o\colon \mathsf{Obs}^\bullet\;\Gamma) \times S^-\;(\Gamma,\Delta + \mathsf{holes}(o))\right)$$
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The "operational strategy"

$$S^+(\Gamma,\Delta) \coloneqq (c\colon \mathsf{Conf}\;\Gamma) \times (\gamma\colon \Delta \to_{\mathsf{Val}} \Gamma)$$
$$S^-(\Gamma,\Delta) \coloneqq \Delta \to_{\mathsf{Val}} \Gamma$$
$$\mathsf{play} \coloneqq \text{“eval then hide arguments”}$$
$$\mathsf{coplay} \coloneqq \text{“apply observation”}$$

Levy & Staton: Transition systems over games

Xia *et al.*: Interaction trees

Composition and the *mystery* hypothesis

Why composition?

OGS soundness in a nutshell

1. Composition respects bisimilarity: congruence.
2. Composition simulates substitution: adequacy.

Given $[\![c]\!] \approx [\![d]\!]$, for any γ,

$$
\begin{aligned}
\mathrm{eval}(c[\gamma]) &\approx [\![c]\!] \parallel [\![\gamma]\!] && \text{(by 2)} \\
&\approx [\![d]\!] \parallel [\![\gamma]\!] && \text{(by 1)} \\
&\approx \mathrm{eval}(d[\gamma]) && \text{(by 2)}
\end{aligned}
$$

Characterizing composition

$$-\|- : \forall \Phi,\ S^{+}\ \Phi \to S^{-}\ \Phi \to \mathcal{D}(\mathsf{Res})$$

$$(c, \gamma) \parallel \delta \;\coloneqq\; \mathsf{let}\ x{\cdot}o(\varphi) \leftarrow \mathsf{eval}(c)\,;\ \mathsf{case}\ x \begin{cases} \mathsf{final} & \mapsto \mathsf{ret}(x{\cdot}o) \\ \mathsf{shared} & \mapsto (\delta(x){\cdot}o(\mathsf{fresh}), \delta) \parallel (\gamma + \varphi) \end{cases}$$

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This is not a coinductive definition 

Chattering, *or, why everything always falls apart*

A bad looping example

$$(c, \gamma) \parallel \delta \qquad \Gamma, \Delta \coloneqq [\neg\mathsf{Bool}]$$

$$c \coloneqq \langle \mathsf{true} \parallel x \rangle \qquad \gamma \coloneqq y \mapsto x \qquad \delta \coloneqq x \mapsto y$$

🚨 Looping without ever doing a reduction step. 🚨

Chattering, *or, why everything always falls apart*

A bad looping example

$$(c, \gamma) \parallel \delta \qquad \Gamma, \Delta := [\neg \mathrm{Bool}]$$

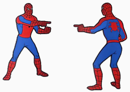
$$c := \langle \mathrm{true} \parallel x \rangle \qquad \gamma := y \mapsto x \qquad \delta := x \mapsto y$$

🚨 Looping without ever doing a reduction step. 🚨

Two live processes sharing a channel

Fine: Stop interacting with the world.

Not Fine: Pointing fingers.



Revisiting OGS positions

An order on variables

Arguments should only mention *older* variables.

Γ and Δ should form an acyclic bipartite graph.

Better types

An interleaving of the two scopes: $\Phi \coloneqq \Gamma_0, \Delta_0, \Gamma_1, \Delta_1, \ldots$

$$\text{my}(\Phi) \coloneqq \Gamma_0, \Gamma_1, \ldots$$

$$\text{your}(\Phi) \coloneqq \Delta_0, \Delta_1, \ldots$$

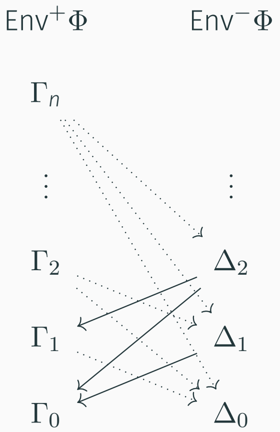
Revisiting assignments

Replace $\Gamma \to_{\mathsf{Val}} \Delta$ with some funny pair of mutual inductives.

Refine OGS states.

$S^{+}\ \Phi := \mathsf{Conf}\ (\mathsf{my}\ \Phi) \times \mathsf{Env}^{+}\ \Phi$

$S^{-}\ \Phi := \mathsf{Env}^{-}\ \Phi$



Finite chattering

Eventually, either

1. Interaction ends.
2. Some (head) variable is replaced by a non-variable value.

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Not enough!

$$\langle\, x \,\|\, y \,\rangle \xrightarrow{chatter} \langle\, x \,\|\, \cdot\mathrm{app}(true, z) \,\rangle \xrightarrow{chatter} \langle\, \lambda a.t \,\|\, \cdot\mathrm{app}(true, z) \,\rangle$$

Two chatters for a redex.

Repeatedly instanciating the head variable of a normal-form by a non-variable value eventually leads to a redex.

$$- \triangleright - : \mathrm{Obs} \to \mathrm{Obs} \to \mathrm{Prop} \qquad \frac{\mathrm{eval}\ (v \cdot o_1(\gamma)) \approxeq \mathrm{ret}\ (x \cdot o_2(\delta))}{o_1 \triangleright o_2}$$

"Finite redexes"

$- \triangleright -$ is well-founded.

Concluding with eventual guardedness

Eventual guardedness

Recursive equations

$$e \colon \overbrace{X \to \mathcal{D}(X}^{\text{variables}} + Y)$$

$$e \colon X \to \mathcal{D}(\underbrace{X}_{\text{recursive calls}} + \underbrace{Y}_{\text{constants}})$$

Guardedness criteria

$e\ x$ is guarded if $e\ x \neq \mathrm{ret}(\mathrm{inl}(x))$.

$e\ x$ is eventually guarded if there exists an n such that $e^n\ x$ is guarded.

Pointwise (eventually) guarded equations admit unique fixpoints (w.r.t. strong bisimilarity).

Conclusion

Contributions

- Formalized generic soundness theorem for OGS.
- OGS for several $\mu\tilde{\mu}$ and λ-calculi.
- New? interesting family of guard conditions.

Future work ideas

- Coq: more flexible language interface.
- Expand: effectful languages.
- Adjacent: completeness, normal form bisimulations.